higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T870(E)(N21)T

NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

21 November 2016 (X-Paper)
09:00–12:00

Nonprogrammable scientific calculators may be used.

This question paper consists of 6 pages and 1 formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order but subsections of questions must NOT be separated.
5. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
6. If necessary, answers should be rounded off to THREE decimals, unless stated otherwise.
7. Diagrams are NOT drawn to scale.
8. Write neatly and legibly.

QUESTION 1

1.1 Simplify:

1.1.1 $\sqrt{49x^2+56x+16}$ (2)

1.1.2 $\left(a^{\frac{1}{3}}-a^{-\frac{2}{3}}\right)(a+a^{-1}-2)^{-1}$ (7)

1.1.3 $\dfrac{3x^2-7x+2}{2x^2-5x-3}\div\dfrac{x^2+x-6}{x^2-9}$ (6)

1.2 If $16x^3-px^2+12x+3$ is divided by $2x+1$, the remainder is -7. Determine the value of p. (4)

[19]

QUESTION 2

2.1 Express $\sqrt{0,125}$ as a power of 4. (3)

2.2 Solve for x:

2.2.1 $2\log x+\log\dfrac{3}{4}-\log\left(2x+\dfrac{3}{4}\right)=0$

Show All the steps. The use of a calculator is excluded. (6)

2.2.2 $\dfrac{4}{x-2}+\dfrac{2x-3}{4-x^2}=\dfrac{5}{x+2}$ (5)

[14]

QUESTION 3

3.1 Solve for x by completing the square: $-18-x^2=9x$ (5)

3.2 There are 302 residents in a town. There are 34 more women than men and 60 more children than women. How many women are there? (3)

3.3 Given: $PV^n=C$
Calculate the value of n if P=2000, V=0,48 and C=18,3. (5)

[13]

QUESTION 4

Consider FIGURE 1 below. ABCD is a parallelogram with vertices A(-2;1), B (2;2), C(1;-2) and D $(x; y)$.

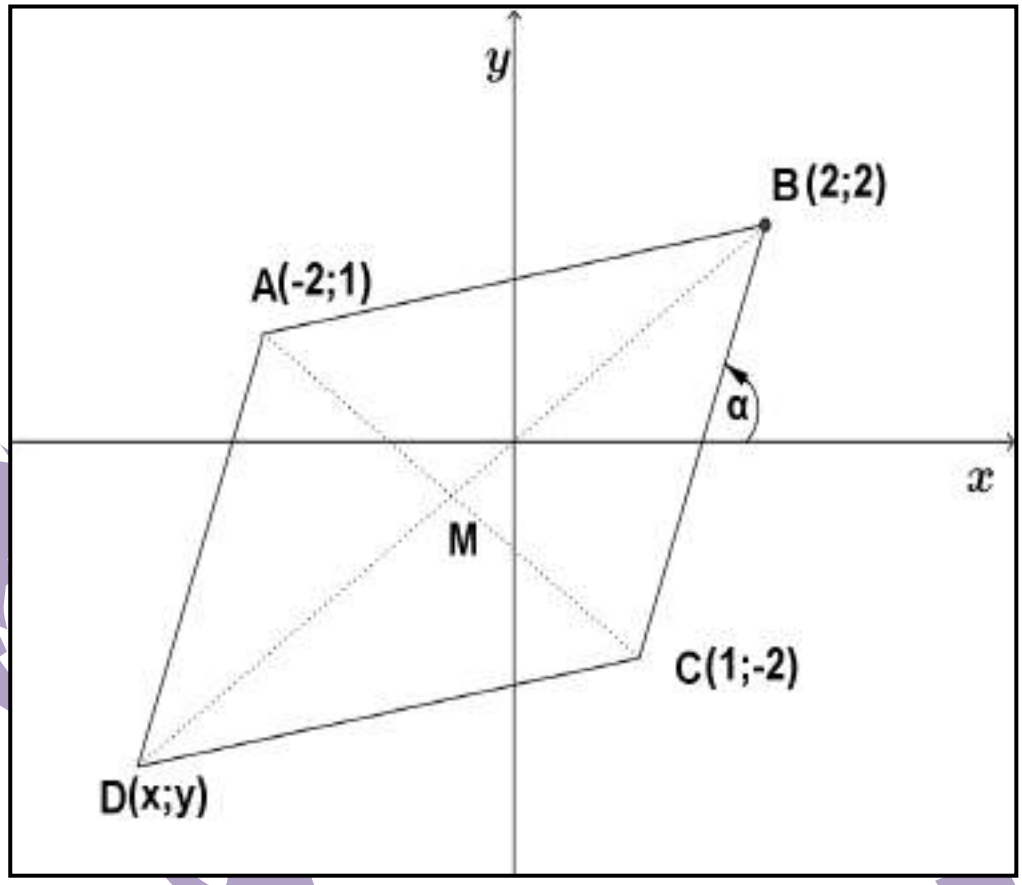

FIGURE 1

4.1 Prove that the midpoint M, of AC is equal to $(-\frac{1}{2};-\frac{1}{2})$. (2)

4.2 Use the co-ordinates of the midpoint, M to determine the co-ordinates of the fourth vertex, D. (3)

4.3 Determine the length of BC. (2)

4.4 Calculate the angle of inclination, α of the line BC. (2)

4.5 Determine the equation of the line in the form $y = mx + c$ which is perpendicular to BC and passes through A. (3)

4.6 Determine the coordinates of the point of intersection of the line through A(-2;1), which is parallel to the y-axis and the line $2y - x = 5$. (3)

[15]

QUESTION 5

5.1 Sketch the graph of $y = -\sqrt{36 - x^2}$ in the ANSWER BOOK. ALL values at the points of intersection with the axes must be shown. (3)

5.2 FIGURE 2 shows the graph of $f(x) = -x^3 + ax^2 + bx + c$.
A and B(6;0) are turning points.

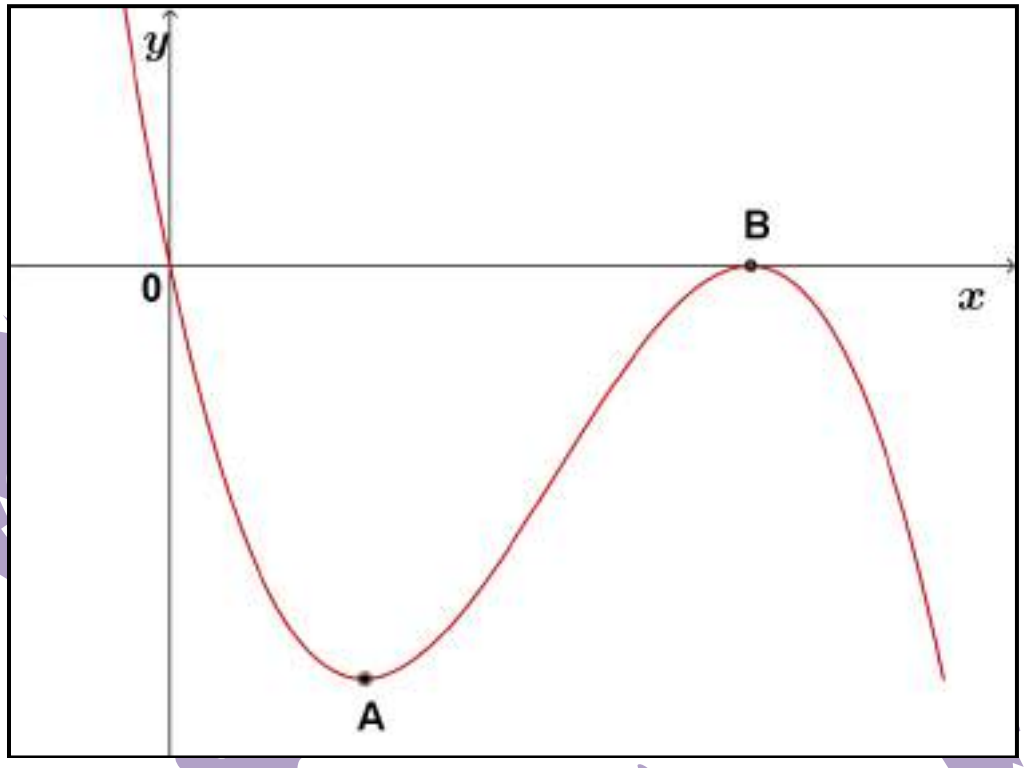

FIGURE 2

5.2.1 Find the values of a, b and c. Show ALL your calculations. (5)

5.2.2 If $f(x) = -x^3 + 12x^2 - 36x$, find the coordinates of A. (4)

5.3 Determine $\frac{dy}{dx}$ if $y = x^{\frac{1}{2}} - \frac{3}{x^2}$ by making use of the rules of differentiation. ALL final answers must be with positive exponents and in surd form where applicable. (4)
[16]

QUESTION 6

6.1 Make use of trigonometric identities to prove the following:

$$\frac{\cos^2(90^\circ + x)}{\sin(90^\circ - x) + 1 - \sin^2 x} = \frac{1 - \cos x}{\cos x}$$ (7)

6.2 To find the height of a tower, a surveyor sets up his theodolite 150 m from the base of the tower. He finds that the angle of elevation to the top of the tower is 60°.
What is the height of the tower? (3)

6.3 Consider FIGURE 3 below. The crew in a boat, C noticed two lighthouses A and B, one on either side of a beach. The two lighthouses are 0,67 km apart and one is exactly due east of the other. The lighthouses tell how close a boat is by taking bearings to the boat. The bearing from the lighthouse A to the boat C is 127° and the bearing from the lighthouse B to the boat C is 255°. A bearing is an angle measured clockwise from north.

Calculate how far the boat, C is from the lighthouse at B. (5)

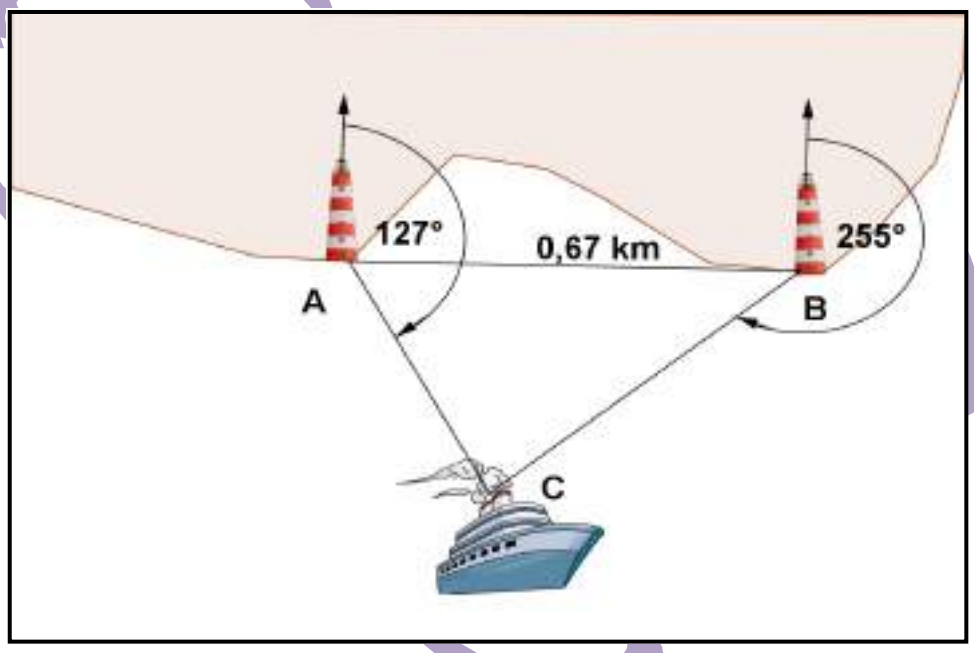

FIGURE 3

6.4 6.4.1 Draw the graphs which are represented by $f(x) = 2\sin 2x$ and $g(x) = \cos 2x$ on the same system of axes for $0° \le x \le 180°$. ALL values at the points of intersection with the system of axes and the coordinates of the turning points must be shown. (6)

6.4.2 Use the graph to read off the value(s) of x if $2\tan 2x = 1$ for $0° \le x \le 180°$ (2)

[23]

TOTAL: 100

FORMULA SHEET

1. Factors

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

2. Logarithms

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \ , \ \therefore \ln e = 1$$

$$a^{\log_a t} = t \ , \ \therefore e^{\ln m} = m$$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. Circle

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

6. Straight line

$$y - y_1 = m(x - x_1)$$

Perpendicular lines : $m_1 \times m_2 = -1$

Parallel lines : $m_1 = m_2$

Distance : $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint : $M = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination : $\theta = \tan^{-1} m$

7. Differentiation

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

For turning point : $f'(x) = 0$

8. Trigonometry

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area of $\Delta ABC = \frac{1}{2} ac \sin B$