higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N3

21 NOVEMBER 2016

This marking guideline consists of 10 pages.

✓ = 1 Mark

QUESTION 1

1.1 1.1.1

$$\sqrt{49x^2+56x+16}$$
$$=\sqrt{(7x+4)^2} \quad ✓$$
$$=7x+4 \quad ✓ \qquad (2)$$

1.1.2

$$\left(a^{\frac{1}{3}}-a^{-\frac{2}{3}}\right)(a+a^{-1}-2)^{-1}$$
$$=\left(\sqrt[3]{a}-\frac{1}{\sqrt[3]{a^2}}\right)(a+\frac{1}{a}-2)^{-1} \quad ✓ \quad ✓$$
$$=\left(\frac{\sqrt[3]{a}.\sqrt[3]{a^2}-1}{\sqrt[3]{a^2}}\right)\left(\frac{a^2-2a+1}{a}\right)^{-1} \quad ✓$$
$$=\left(\frac{a-1}{\sqrt[3]{a^2}}\right)\left(\frac{(a-1)^2}{a}\right)^{-1} \quad ✓ \quad ✓$$
$$=\left(\frac{a-1}{a^{\frac{2}{3}}}\right)\frac{a}{(a-1)^2} \quad ✓$$
$$=\frac{a^{\frac{1}{3}}}{(a-1)} or \frac{\sqrt[3]{a}}{a-1} \quad ✓$$

$$\left(a^{\frac{1}{3}}-a^{-\frac{2}{3}}\right)(a+a^{-1}-2)^{-1}$$
$$=\left(a^{\frac{1}{3}}-\frac{1}{a^{\frac{2}{3}}}\right)(a+\frac{1}{a^1}-2)^{-1} \quad ✓ \quad ✓$$
$$=\left(\frac{a-1}{a^{\frac{2}{3}}}\right)\left(\frac{a^2-2a+1}{a}\right)^{-1} \quad ✓ \quad ✓$$
$$=\left(\frac{a-1}{a^{\frac{2}{3}}}\right)\frac{a}{(a-1)^2} \quad ✓ \quad ✓$$
$$=\frac{a^{\frac{1}{3}}}{(a-1)} \quad ✓$$

(7)

1.1.3

$$\frac{3x^2-7x+2}{2x^2-5x-3}\div\frac{x^2+x-6}{x^2-9}$$
$$\frac{(3x-1)(x-2)}{(2x+1)(x-3)} \quad ✓ \times ✓ \frac{(x-3)(x+3)}{(x-2)(x+3)} \quad ✓ \quad ✓$$
$$=\frac{3x-1}{2x+1} \quad ✓ \qquad (6)$$

1.2 If $f(x)$ is divided by 2x+1, then the remainder is $f\left(-\frac{1}{2}\right)$ ✓

Hence $f\left(-\frac{1}{2}\right)=16\left(\frac{-1}{2}\right)^3-p\left(-\frac{1}{2}\right)^2+12\left(-\frac{1}{2}\right)+3=-7$ ✓

$$16\left(-\frac{1}{8}\right)-\frac{1}{4}p-6+3=-7$$

$$-2-\frac{1}{4}p-6+3=-7$$

$$-\frac{1}{4}p=-7+5 \quad ✓$$

$$p=-2\times-\frac{4}{1}$$

$$p=8 \ ✓$$

(4)
[19]

QUESTION 2

2.1 $\sqrt{0,125}$

$=\left(\frac{1}{8}\right)^{\frac{1}{2}}$ ✓

$=\left(\frac{1}{2^3}\right)^{\frac{1}{2}}$ ✓

$=2^{-\frac{3}{2}}=4^{-\frac{3}{4}}$ ✓

$\sqrt{0,125}=4^x$

$\therefore\sqrt{\frac{1}{8}}=4^x$ ✓

$\therefore\sqrt{2^{-3}}=2^{2x}$ ✓

$\therefore-\frac{3}{2}=2x$

$\therefore x=-\frac{3}{4}$ ✓

$\therefore\sqrt{0,125}=4^{-\frac{3}{4}}$

(3)

2.2 2.2.1

$$2\log x + \log\frac{3}{4} - \log\left(2x+\frac{3}{4}\right) = 0$$

$$\log_{10}\frac{\frac{3}{4}x^2}{2x+\frac{3}{4}} = 0 \quad \checkmark$$

$$10^0 = \frac{\frac{3}{4}x^2}{2x+\frac{3}{4}} \quad \checkmark$$

$$\frac{3x^2}{8x+3} = 1 \quad \checkmark$$

$$3x^2 = 8x+3 \quad \checkmark$$

$$3x^2 - 8x - 3 = 0 \quad \checkmark$$

$$(3x+1)(x-3) = 0$$

$$3x+1=0 \qquad x-3=0$$

$$x = \frac{-1}{3}\text{(n/a)} \qquad x = 3 \quad \checkmark$$

$$2\log x + \log\frac{3}{4} - \log\left(2x+\frac{3}{4}\right) = 0$$

$$\therefore \log x^2 + \log\frac{3}{4} = \log\left(2x+\frac{3}{4}\right) \quad \checkmark$$

$$\therefore \log\left(\frac{3x^2}{4}\right) = \log\left(2x+\frac{3}{4}\right) \quad \checkmark$$

$$\therefore \frac{3x^2}{4} = 2x+\frac{3}{4} \quad \checkmark$$

$$\therefore 3x^2 = 8x+3 \quad \checkmark$$

$$\therefore 3x^2 - 8x - 3 = 0 \quad \checkmark$$

$$\therefore (3x+1)(x-3) = 0$$

$$\therefore 3x+1=0 \qquad x-3=0$$

$$\therefore x = \frac{-1}{3}\text{(n/a)} \qquad x = 3 \quad \checkmark$$

(6)

2.2.2

$$\frac{4}{x-2} + \frac{2x-3}{4-x^2} = \frac{5}{x+2}$$

$$\frac{4}{x-2} - \frac{2x-3}{(x-2)(x+2)} = \frac{5}{x+2} \quad \checkmark \quad \checkmark$$

$$4(x+2) - 2x + 3 = 5(x-2) \quad \checkmark$$

$$4x+8-2x+3 = 5x-10$$

$$-3x = -21 \quad \checkmark$$

$$x = 7 \quad \checkmark$$

(5)
[14]

QUESTION 3

3.1

$-18-x^2=9x$

$x^2+9x+18=0$ ✓

$x^2+9x+\frac{81}{4}=-18+\frac{81}{4}$ ✓

$\therefore\left(x+\frac{9}{2}\right)^2=2,25$ ✓

$x=-\frac{9}{2}\pm\sqrt{2,25}$

$x=-3 \quad or \quad x=-6$ ✓ ✓ (5)

3.2

Let the number of women = x

$\therefore$ the number of men = $x-34$ ✓

and the number of children = $x+60$

$\therefore x+x-34+x+60=302$ ✓

$\therefore 3x=276$

$\therefore x=92$

There are 92 women ✓

Let the number of men = x

let the number of women = $x+34$ ✓

let the number of children = $x+34+60$

$\therefore x+x+34+x+94=302$ ✓

$3x+128=302$

$3x=174$

$x=58$

There are 92 women ✓ (3)

3.3

$PV^n=C$

$V^n=\frac{C}{P}$ ✓

$\log V^n=\log\frac{C}{P}$ ✓

$n\log V=\log C-\log P$ ✓

$n=\frac{\log C-\log P}{\log V}$

$n=\frac{\log 18,3-\log 2000}{\log 0,48}$ ✓

$n=\frac{1,262-3,301}{-0,319}$

$n=6,395$ ✓

$PV^n=C$

$\therefore 2000\times(0,48)^n=18,3$ ✓

$\therefore (0,48)^n=\frac{18,3}{2000}$ ✓

$\therefore \log(0,48)^n=\log\left(\frac{18,3}{2000}\right)$ ✓

$\therefore n\log(0,48)=\log\left(\frac{18,3}{2000}\right)$

$\therefore n=\frac{\log 18,3-\log 2000}{\log 0,48}$ ✓

$\therefore n=\frac{1,262-3,301}{-0,319}$

$n=6,395$ ✓ (5)

[13]

QUESTION 4

4.1
$$M(x_M;y_M)=\left(\frac{-2+1}{2};\frac{1-2}{2}\right)\checkmark$$
$$=\left(\frac{-1}{2};\frac{-1}{2}\right)\quad\checkmark$$
(2)

4. 2 $DM = MB$ ..... diagonals of parm

Midpoint of $DB = M(x_M;y_M)=\left(\frac{-1}{2};\frac{-1}{2}\right)$

$$\therefore M(x_M;y_M)=\left(\frac{x+2}{2};\frac{y+2}{2}\right)=\left(\frac{-1}{2};\frac{-1}{2}\right)\quad\checkmark$$

$$\therefore \frac{x+2}{2}=\frac{-1}{2}\quad\text{and}\quad\frac{y+2}{2}=\frac{-1}{2}\quad\checkmark$$

$$\therefore x=-3\qquad y=-3$$

Coordinates of $D=(-3;-3)$ ✓
(3)

4.3
$$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{[1-2]^2+[-2-2]^2}$$
$$=\sqrt{1+16}\ \checkmark$$
$$=\sqrt{17}\text{ units}\quad\checkmark$$
(2)

4.4
$$m_{BC}=\frac{2-(-2)}{2-1}=4\quad\checkmark$$
$$\alpha=\tan^{-1}m=\tan^{-1}(4)=75,964^{\circ}\quad\checkmark$$
(2)

4.5 The gradient of a line perpendicular to $BC=-\frac{1}{4}$

The equation of the line is

$$y-y_A=-\frac{1}{4}(x-x_A)$$
$$y-1=-\frac{1}{4}(x+2)\quad\checkmark\quad\checkmark$$
$$y-1=-\frac{1}{4}x-\frac{1}{2}$$
$$\therefore y=-\frac{1}{4}x+\frac{1}{2}\quad\checkmark$$
(3)

4.6 The gradient is not defined when a line is parallel to the y-axis(that is, vertical). ALL x co-ordinates are equal

$\therefore x = -2$ is the equation of the line ……..equation 1 ✓

$2y - x = 5$ …….equation 2

Solve equation 1 and 2 simultaneously

Substitute 1 into 2

$\therefore 2y - (-2) = 5$

$y = \frac{3}{2}$ ✓

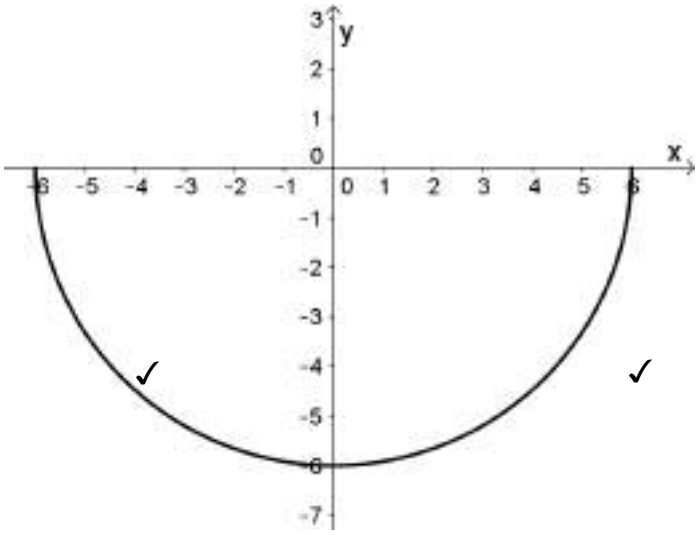
Point of intersection $(-2; \frac{3}{2})$ ✓

(3)

[15]

QUESTION 5

5.1 $y = -\sqrt{36 - x^2}$ ✓



✓ ✓

1 mark for scale
2 marks for correct graph

(3)

5.2 5.2.1 Since the zeros of $f(x)$ are 0, 6 and 6 (the x-axis is tangent to the curve):

$f(x) = -x(x-6)(x-6)$ ✓ Coefficient of x^3 is -1 as given.

$f(x) = -x^3 + 12x^2 - 36x$ ✓

$a = 12, b = -36$ and c=0 ✓ ✓ ✓

(5)

5.2.2 $f'(x) = -3x^2 + 24x - 36$ ✓

but at the turning point , $f'(x) = 0$

$\therefore -3x^2 + 24x - 36 = 0$ ✓

$\therefore x^2 - 8x + 12 = 0$ ✓

$\therefore (x-6)(x-2) = 0$

$\therefore x = 6 \quad \text{or } x = 2$ ✓

If $x = 2$ then $f(2) = -(2)^3 + 12(2)^2 - 36(2) = -32$

$\therefore A(2;-32)$ (4)

5.3 $y = x^{\frac{1}{2}} - \frac{3}{x^2}$

$y = x^{\frac{1}{2}} - 3x^{-2}$ ✓

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + 6x^{-3}$ ✓ ✓

$= \frac{1}{2\sqrt{x}} + \frac{6}{x^3}$ ✓

(4)
[16]

QUESTION 6

6.1 $\frac{\cos^2(90^\circ + x)}{\sin(90^\circ - x) + 1 - \sin^2 x} = \frac{1 - \cos x}{\cos x}$

$LHS = \frac{\cos^2(90^\circ + x)}{\sin(90^\circ - x) + 1 - \sin^2 x}$

$= \frac{\sin^2 x}{\cos x + \cos^2 x}$ ✓ ✓ ✓

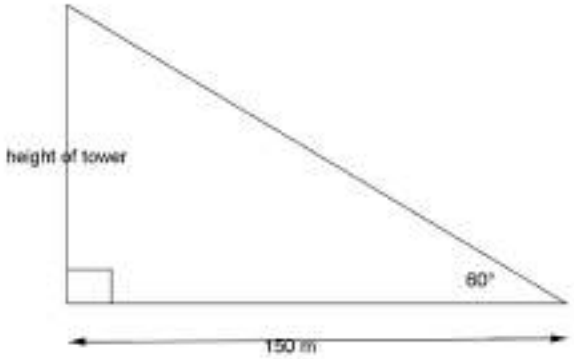
$= \frac{1 - \cos^2 x}{\cos x(1 + \cos x)}$ ✓ ✓

$= \frac{(1 - \cos x)(1 + \cos x)}{\cos x(1 + \cos x)}$ ✓

$= \frac{1 - \cos x}{\cos x}$ ✓

$= RHS$ (7)

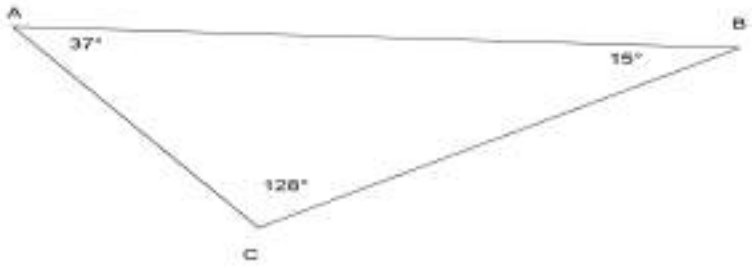
6.2



$$\tan 60^\circ = \frac{\text{height of tower}}{150}$$

$$\text{height of tower} = 150 \times \tan 60^\circ = 259,808 \text{ m} \quad ✓ ✓$$ (3)

6.3



6.3.1

$$\hat{A} = 127^\circ - 90^\circ = 37^\circ \quad ✓$$

$$\hat{B} = 270^\circ - 255^\circ = 15^\circ \quad ✓$$

$$\hat{C} = 180^\circ - 15^\circ - 37^\circ = 128^\circ$$

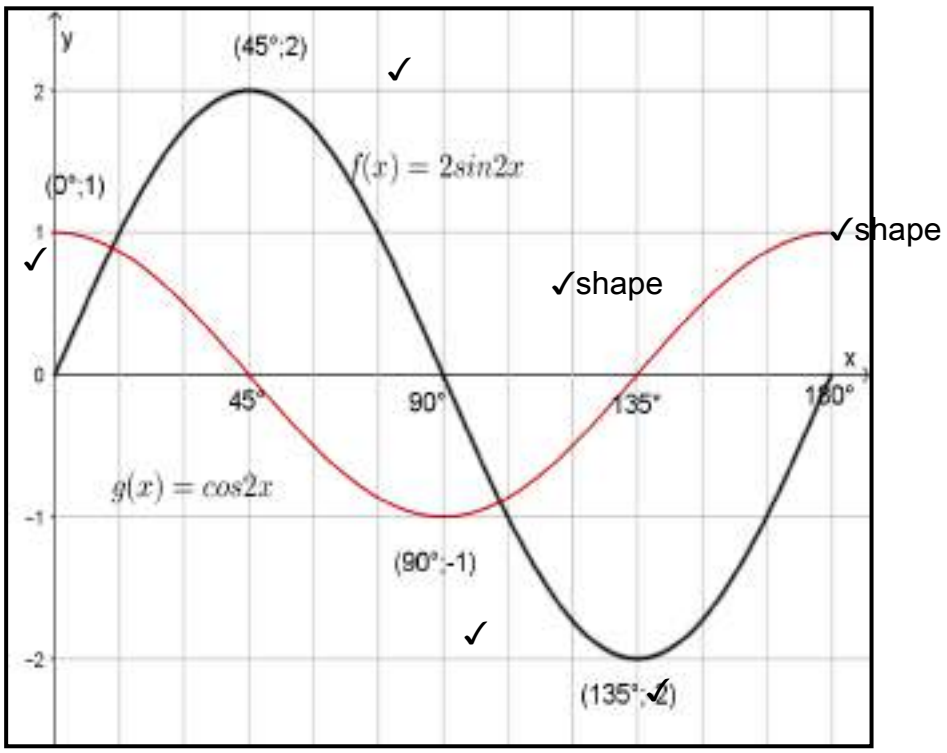
$$\frac{BC}{\sin A} = \frac{AB}{\sin C} \quad ✓$$

$$\frac{BC}{\sin 37^\circ} = \frac{0,67}{\sin 128^\circ} \quad ✓$$

$$BC = \frac{0,67 \times \sin 37^\circ}{\sin 128^\circ}$$

$$= 0,51 \text{ km} \quad ✓$$ (5)

6.4 6.4.1



Shape(1 each = 2) Min TP (1 each = 2) Max TP (1 each = 2) = 6 (6)

6.4.2 13° and 103° ✓ ✓ (2)

[23]

TOTAL: 100