higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

STRENGTH OF MATERIALS AND STRUCTURES N6

8 APRIL 2016

This marking guideline consists of 8 pages.

QUESTION 1

1.1

$$at\ 100\ mm: a - \frac{b}{0{,}1^2} = 180 \times 10^6 \ldots\ldots\ldots (1)\checkmark (inner\ cylinder)$$

$$at\ 100\ mm: a + \frac{b}{0{,}1^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)\checkmark$$

$$(1) + (2): \ 2a = 180 \times 10^6$$

$$a = 90 \times 10^6 \checkmark$$

$$b = 900 \times 10^3 \checkmark$$

$$at\ 200\ mm: \ \sigma_R = a + \frac{b}{0{,}2^2}$$

$$= 90 \times 10^6 + \frac{900 \times 10^3}{0{,}2^2}$$

$$\sigma_R = 67{,}5\ MPa \ \checkmark$$

$$at\ 300\ mm: a + \frac{b}{0{,}3^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)\checkmark (outer\ cylinder)$$

$$at\ 200\ mm: a - \frac{b}{0{,}2^2} = -200 \times 10^6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)\checkmark$$

$$(1) - (2): \ 11{,}111b + 25b = 200 \times 10^6$$

$$b = 5{,}538 \times 10^6 \checkmark$$

$$a = -61{,}538 \times 10^3 \checkmark$$

$$at\ 200\ mm: \ \sigma_R = a + \frac{b}{0{,}2^2}$$

$$= -61{,}538 \times 10^6 + \frac{5{,}538 \times 10^6}{0{,}2^2}$$

$$\sigma_R = 76{,}923\ MPa \ \checkmark$$

$$Therefore\ the\ maximum\ pressure = 67{,}5\ MPa \ \checkmark$$ (11)

1.2 $$\delta d_1 = \frac{d}{E}(\sigma_H - \vartheta \times \sigma_R)$$

$$= \frac{0,1}{200 \times 10^9}(180 \times 10^6 - 0,29 \times 0) \checkmark$$

$$\delta d_1 = 90 \times 10^{-6}\ m \checkmark$$

(2)
[13]

QUESTION 2

2.1 *Consider stress limit:*

$$M = \sigma \times Z = 180 \times 10^6 \times 1329 \times 10^{-6} = 239,22\ kNm \checkmark$$

$$w = \frac{8M}{L^2} = \frac{8 \times 239,22 \times 10^3}{4^2} \checkmark = 119,61\ kNm \checkmark$$

Consider deflection limit:

$$w = \frac{384EIy}{5L^4} = \frac{384 \times 200 \times 10^9 \times 274,2 \times 10^{-6} \times 0,011}{5 \times 4^4} \checkmark = 180,972\ kNm \checkmark$$

The maximum allowed weight = total weight − own weight

$$w = 119,61 \times 10^3 - 74,8 \times 9,81 \checkmark = 118,876\ kNm \checkmark$$

(7)

2.2 *actual stress* $\sigma_b = 180\ MPa \checkmark$

$$y = \frac{5wL^4}{384EI} = \frac{5 \times 119,61 \times 10^3 \times 4^4}{384 \times 200 \times 10^9 \times 274,2 \times 10^{-6}} \checkmark = 7,27 \times 10^{-3}\ m \checkmark$$

(3)

2.3 $$y = \frac{FL^3}{48EI}$$

$$(7,27 - 3) \times 10^{-3} \checkmark = \frac{F \times 4^3}{48 \times 200 \times 10^9 \times 274,2 \times 10^{-6}} \checkmark$$

$$F = 175,635\ kNm \checkmark$$

(3)
[13]

QUESTION 3

3.1

$$\sigma_d = \frac{F}{A} = \frac{60 \times 10^3}{5{,}876 \times 10^{-6}} \checkmark = 10{,}211\ MPa \checkmark$$

$$M = \frac{Fab}{L} = \frac{40 \times 10^3 \times 1 \times 3}{4} = 30\ kNm \checkmark$$

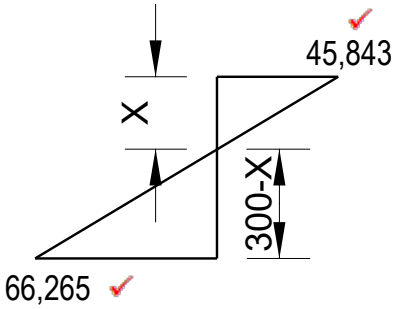
$$\sigma_b = \frac{MY}{I} = \frac{30 \times 10^3 \times 0{,}15}{80{,}28 \times 10^{-6}} \checkmark = 56{,}0538\ MPa \checkmark$$

$$\sigma_{bottom} = \sigma_d + \sigma_b = 10{,}211 + 56{,}0538 = 66{,}265\ MPa \checkmark (tensile) \checkmark$$

$$\sigma_{top} = \sigma_d - \sigma_b = 10{,}211 - 56{,}0538 = 45{,}843\ MPa \checkmark (compressive) \checkmark$$

(9)

3.2



$$\frac{45{,}843}{x} = \frac{66{,}265}{300 - x} \checkmark$$

$$x = 122{,}675\ mm\ (from\ top) \checkmark$$

(4)
[13]

QUESTION 4

4.1

$$W = \rho g A l = 2400 \times 9{,}81 \times 0{,}5 \times B \times 5 \times 1 = 58860B = V \checkmark$$

$$F_w = \frac{\rho g h^2}{2} = \frac{1000 \times 9{,}81 \times 5^2}{2} = 122625\ N \checkmark$$

$$Moments\ about\ the\ toe:\ V \times x_R + F_w \times \frac{h}{3} = W \times x_1$$

$$58860B \times \frac{B}{3} \checkmark + 122625 \times \frac{5}{3} \checkmark = 58860B \times \frac{2B}{3} \checkmark$$

$$B = 3{,}227\ m \checkmark$$

(6)

4.2

$$W{\sim}M = W \times x_1 = 58860 \times 3{,}227 \times 0{,}667 \times 3{,}227 = 408{,}627\ kNm \checkmark$$

$$F{\sim}M = F_w \times \frac{h}{3} = 122625 \times \frac{5}{3} = 204{,}375\ kNm \checkmark$$

$$FOS = \frac{W{\sim}M}{F{\sim}M} = \frac{408{,}627}{204{,}375} = 2 \checkmark (safe) \geq 2 \checkmark$$

(4)

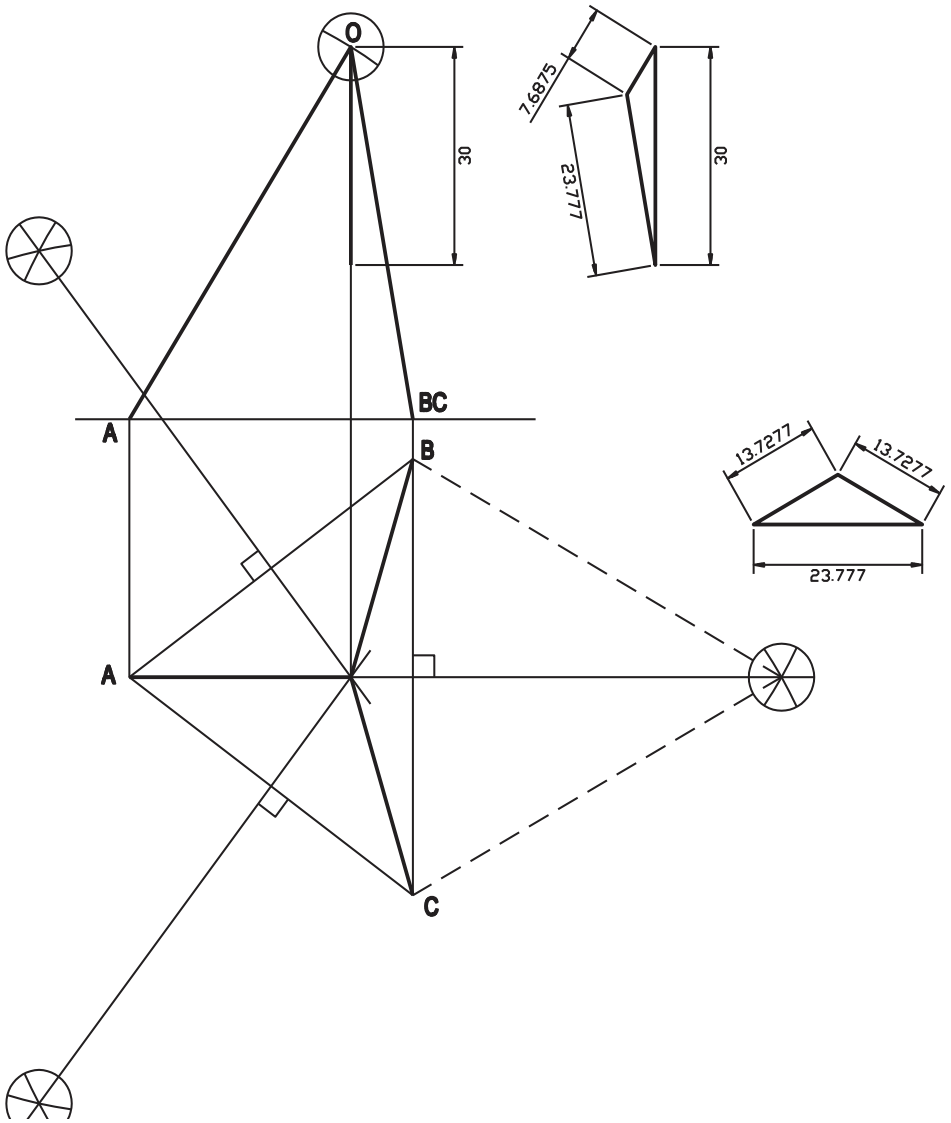
4.3 $V = 58860B = 58860 \times 3{,}227 = 189{,}941\ kN$ ✓

$$FOS = \frac{\mu V}{F_w} = \frac{0{,}4 \times 189{,}941}{122{,}625} = 0{,}62\ \checkmark (not\ safe) < 1{,}5\ \checkmark$$

(3)
[13]

QUESTION 5

5.1



(2)

5.2

MEMBER	MAGNITUDE	NATURE
OA	7,68 kN ✓	Strut
OB	13,73 kN ✓	Strut
OC	13,73 kN ✓	Strut

(7)
[9]

QUESTION 6

6.1 $$mA_s(d-n) = A_1y_1 + A_2y_2$$

$$15 \times 200 \times 10^{-6}(0{,}6-n) \checkmark = 0{,}5 \times 0{,}08(n-0{,}04) \checkmark + 0{,}2 \times 0{,}5(n-0{,}08)^2 \checkmark$$

$$n^2 + 270n - 27600 = 0$$

$$n = 0{,}0791\ m \checkmark \quad (4)$$

6.2 $$l_a = d - \frac{n}{3} = 0{,}6 - \frac{0{,}0791}{3} = 0{,}574\ m \checkmark$$

$$M = 0{,}5\sigma_c A_c l_a = 0{,}5 \times 5{,}2 \times 10^6 \times 0{,}5 \times 0{,}0791 \times 0{,}574 = 58{,}964\ kNm \checkmark$$

$$M = \sigma_s A_s l_a = 140 \times 10^6 \times 200 \times 10^{-6} \times 0{,}574 = 16{,}062\ kNm \checkmark$$

$$Therefore\ maximum\ M = 16{,}062\ kNm \checkmark\checkmark \quad (5)$$

6.3 $$M = 0{,}5\sigma_c A_c l_a$$

$$16{,}062 \times 10^3 = 0{,}5 \times \sigma_c \times 0{,}5 \times 0{,}0791 \times 0{,}574 \checkmark$$

$$\sigma_c = 1{,}417\ MPa \checkmark \quad (2)$$

6.4 $$M_c = 0{,}5\sigma_c A_c \frac{2}{3}n$$

$$= 0{,}5 \times 1{,}417 \times 10^6 \times 0{,}5 \times 0{,}0791 \times \frac{2}{3} \times 0{,}0791 \checkmark$$

$$M_c = 1{,}477\ kNm \checkmark \quad (2)$$

6.5 $$M_s = \sigma_s A_s (d-n)$$

$$= 140 \times 10^6 \times 200 \times 10^{-6}(0{,}6 - 0{,}0791) \checkmark$$

$$M_s = 14{,}586\ kNm \checkmark \quad (2)$$

[15]

QUESTION 7

7.1

$$y_0 = \frac{F_H}{w} = \frac{250}{10} = 25\ m \checkmark$$

$$y_1 = y_0 + d = 25 + 6 = 31\ m \checkmark$$

$$F_{T1} = wy_1 = 10 \times 31 = 310\ N \checkmark$$

$$y_2 = y_1 + h = 31 + 7 = 38\ m \checkmark$$

$$F_{T2} = wy_2 = 10 \times 38 = 380\ N \checkmark$$ (5)

7.2

$$l_1 = \sqrt{y_1^2 - y_0^2} = \sqrt{31^2 - 25^2} = 18{,}33\ m \checkmark$$

$$l_2 = \sqrt{y_2^2 - y_0^2} = \sqrt{38^2 - 25^2} = 28{,}618\ m \checkmark$$

$$l_T = l_1 + l_2 = 18{,}33 + 28{,}618\ 46{,}948\ m \checkmark$$ (3)

7.3

$$x_1 = y_0 ln\left(\frac{y_1 + l_1}{y_0}\right) = 25 \times ln\left(\frac{31 + 18{,}33}{25}\right) = 16{,}992\ m \checkmark$$

$$x_2 = y_0 ln\left(\frac{y_2 + l_2}{y_0}\right) = 25 \times ln\left(\frac{38 + 28{,}618}{25}\right) = 24{,}503\ m \checkmark$$

$$x_T = x_1 + x_2 = 16{,}992 + 24{,}503 = 41{,}494\ m \checkmark$$ (3)

7.4

$$y_3 = \frac{F_{H3}}{w} = \frac{350}{10} = 35\ m \checkmark$$

$$l_3 = \sqrt{y_3^2 - y_0^2} = \sqrt{35^2 - 25^2} = 24{,}495\ m \checkmark$$

$$x_3 = y_0 ln\left(\frac{y_3 + l_3}{y_0}\right) = 25 \times ln\left(\frac{35 + 24{,}495}{25}\right) = 21{,}675\ m \checkmark$$ (3)

[14]

QUESTION 8

8.1 $$M_e = \frac{\pi(D^4 - d^4)\sigma}{32D} = \frac{\pi(0{,}1^4 - 0{,}05^4) \times 100 \times 10^6}{32 \times 0{,}1} \checkmark = 9{,}204\ kNm \checkmark$$

8.2 $$M_e = 0{,}5(M + T_e)$$

$$9{,}204 = 0{,}5(5 + T_e) \checkmark$$

$$T_e = 13{,}408\ kNm \checkmark$$

8.3 $$T = \sqrt{T_e^2 - M^2} = \sqrt{13{,}408^2 - 5^2} \checkmark = 12{,}441\ kNm \checkmark$$

8.4 $$\tau = \frac{16DT_e}{\pi(D^4 - d^4)} = \frac{16 \times 0{,}1 \times 13{,}408 \times 10^3}{\pi(0{,}1^4 - 0{,}05^4)} \checkmark = 72{,}839\ MPa \checkmark$$

8.5 $$T_a = \frac{T}{1{,}15} = \frac{12{,}441}{1{,}15} = 10{,}818\ kNm \checkmark$$

$$P = 2\pi NT_a = 2\pi \times 5 \times 10{,}818 = 339{,}854\ kW \checkmark$$

(5 x 2) **[10]**

TOTAL: 100